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Noise reduction using a matching input transformer

J O Lekkala and J A V Malmivuo
Tampere University of Technology, Biomedical Engineering
Laboratory, PO Box 527 33101 Tampere 10, Finland

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Abstract This article describes the noise characteristics of a magnetic field measurement system which consists of an induction coil detector, a matching input transformer and a sensitive preamplifier. The best noise performance is obtained when the amplifier operates with optimum source resistance. The optimal conditions are obtained by using a transformer to match the source and amplifier resistances. The limited frequency response of the transformer, however, often restricts its application. The methods of designing a broad-band instrumentation transformer are discussed. As an example the transformer construction for a magnetocardiographic measurement system and some measurement results are also presented.

1 Introduction

The most simple magnetometer construction is an induction coil. A voltage is induced in the coil when it is placed into a varying magnetic field. This voltage is proportional to the time derivative of the field and after integration we obtain a signal proportional to the magnetic field strength. The difficulty in the construction of this kind of magnetometer is the noise arising in the equipment. The noise can be reduced to the minimum by matching the impedance between the induction coil and the amplifier so that the noise factor is as good as possible and by designing the transformer so that its noise level is low and the pass band is wide. After careful design we can obtain a noise figure of the order of 3 dB. Then the noise arising in the amplifier is equal to the thermal noise arising in the induction coil.

2 Amplifier noise characteristics

It is possible to model the internal noise sources of an amplifier by referring them to its input. This parameter is the so-called equivalent input noise and it refers all the noise sources of the amplifier to the signal source location. The equivalent input noise is (Motchenbacher and Fitchen 1973)

$$\overline{e_{ni}^2}=\overline{e_t^2}+\overline{e_n^2}+\overline{i_n^2}R_s^2+2\gamma\sqrt{\overline{e_n^2}}\,\sqrt{\overline{i_n^2}}\,R_s \qquad (1)$$

where

$\overline{e_t^2}$ = thermal noise voltage of the source (in V^2) ($=4kT\Delta fR_s$),

$\overline{e_n^2}$ = zero impedance noise voltage of the amplifier (in V^2),

$\overline{i_n^2}$ = infinite impedance noise current of the amplifier (in A^2),

R_s = source resistance (in Ω),

γ = correlation coefficient.

The noise characteristics of an amplifier are described by a noise figure. The noise figure NF is determined by

$$\mathrm{NF}=10\lg(\overline{e_{ni}^2}/\overline{e_t^2}). \qquad (2)$$

It tells what fraction of the total noise is generated in the amplifier compared to the total equivalent input noise which includes the source noise and the amplifier noise.

The best operating point is at the minimum of the noise figure. From the definition of the noise figure we can calculate the optimum source resistance R_0 which minimises the noise figure.

$$R_0=(\overline{e_n^2}/\overline{i_n^2})^{1/2}. \qquad (3)$$

Note that the optimum resistance R_0 is *not* necessarily the resistance for maximum power transfer. R_0 is determined by the amplifier noise mechanisms and gives the maximum signal-to-noise ratio. There is no direct relation between R_0 and the amplifier input impedance Z_1. Optimum power transfer ($R_0=Z_1$) maximises only the signal amplitude.

Optimum noise figure ($R_s=R_0$) is given by

$$\mathrm{NF}_0=10\lg[1+(\sqrt{\overline{e_n^2}\,\overline{i_n^2}}/2kT\Delta f)] \qquad (4)$$

where

k = Boltzmann constant (1.3805×10^{-23} J K^{-1}),

T = absolute temperature (in K),

Δf = frequency bandwidth in (Hz).

The following procedure is then used for minimising the noise figure:

(i) we choose an amplifier with $(\overline{e_n^2}\,\overline{i_n^2})^{1/2}$ as small as possible (equation (4));

(ii) we choose $R_s=R_0=(\overline{e_n^2}/\overline{i_n^2})^{1/2}$ (equation (3));

(iii) if the source resistance R_s is not equal to $\sqrt{\overline{e_n^2}/\overline{i_n^2}}$ we can use an impedance transformer which matches the impedance level so that the amplifier sees the optimum source resistance.

Let us discuss the transformer construction in detail. Let the ratio of the turns of the primary and the secondary coils of a transformer be $1:N$. Then the noise referred to the primary of the transformer or the equivalent input noise of the new amplifier system using a transformer is (Nezer 1974)

$$\overline{e_{ni}^2}=\overline{e_t^2}+\overline{e_n^2}/N^2+N^2\overline{i_n^2}R_s^2+2\gamma\sqrt{\overline{e_n^2}}\,\sqrt{\overline{i_n^2}}\,R_s. \qquad (5)$$

From this we can calculate the optimum turn ratio N_0 which gives minimum $\sqrt{\overline{e_{ni}^2}}$

$$N_0=\left(\frac{\sqrt{\overline{e_n^2}}}{\sqrt{\overline{i_n^2}}\,R_s}\right)^{1/2}=\left(\frac{R_0}{R_s}\right)^{1/2}. \qquad (6)$$

The optimum noise figure depends on the product $(\overline{e_n^2}\,\overline{i_n^2})^{1/2}$ of equation (4) which is determined by the characteristics of the amplifier. If $(\overline{e_n^2}\,\overline{i_n^2})^{1/2}$ is small it means for real amplifiers that $\overline{i_n^2}$ is small (FET-input amplifiers). Then the optimum source resistance is high (equation (3)). If $(\overline{e_n^2}\,\overline{i_n^2})^{1/2}$ is large, then the noise figure is not only high but also very sensitive to the changes of the source impedance (equation (4)). When $(\overline{e_n^2}\,\overline{i_n^2})^{1/2}$ is small the term $\overline{i_n^2}R_s^2$ in equation (1) is not significant. For this reason the source resistance can differ from the optimum value and the noise voltage due to the noise current of the amplifier is still negligible (figure 1).

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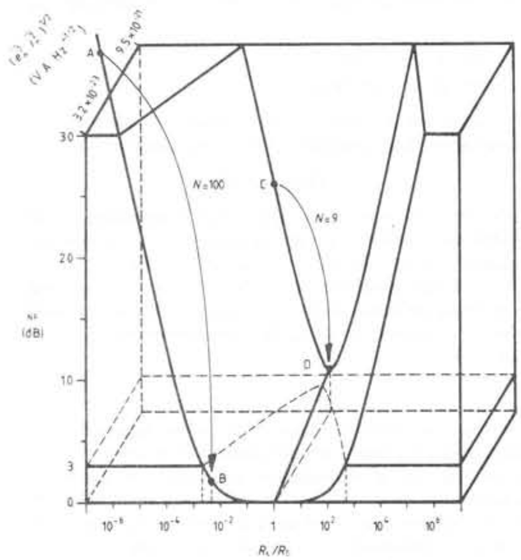

Figure 1 Noise figure against source resistance with two different amplifiers. In FET-input stage amplifier the noise figure with coil detector (point A) can be lowered below 3 dB point by using a transformer with turn ratio $N=100$ (point B). In bipolar-input stage amplifier when using a transformer to get optimum noise figure (from point C to point D), the value of the noise figure in that point is still above 3 dB. The turns ratio of the transformer is then also much lower. This leads to a smaller signal and decreases the signal-to-noise ratio.

In order to obtain the best noise characteristics of the total magnetometer system we must use a coil detector with low source resistance and an amplifier with high input impedance. This requires a matching input transformer with high turn ratio N. However, the use of such a transformer is limited because of its mechanical design due to the large turn ratio.

3 Matching input transformer design

At low frequencies, when the winding shunt capacitances are negligible, the equivalent network of a transformer can be shown as figure 2. In the input transformers the core loss equivalent shunt resistance R_c may be omitted or $R_c=\infty$. At low frequencies the transfer ratio is (ITT 1975)

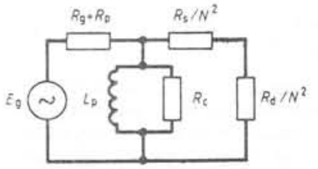

Figure 2 The equivalent network of a transformer at low frequencies. E_g, source generator voltage; R_g, source generator resistance; R_p, primary winding resistance; L_p, primary winding inductance; R_c, coreloss equivalent shunt resistance; R_s, secondary winding resistance; R_d, load resistance; N, secondary-to-primary turn ratio.

$$\frac{E_{\text{out}}}{E_g}=\frac{N}{\{1+[(R_1R_2/(N^2R_1+R_2))\,1/2\pi fL_p]^2\}^{1/2}},\tag{7}$$

where $R_1=R_g+R_p$ and $R_2=R_d+R_s$.

Presuming that $R_s \ll R_d$ and turn ratio is large, we can write the lower cutoff frequency (-3 dB point) (Langford-Smith 1967):

$$f_L=(R_g+R_p)/2\pi L_p.\tag{8}$$

As we see, the lower cutoff frequency decreases when the term R_g+R_p decreases. Another method of decreasing the lower cutoff frequency is to increase the inductance of the primary winding. The inductance of the primary winding will increase when the turn number increases, but then the resistance of the primary also increases. A better method of increasing L_p is to use core material of high permeability.

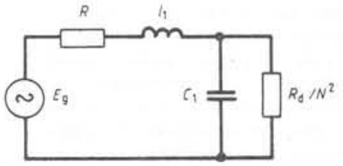

Figure 3 The equivalent network of a transformer at high frequencies. $R=R_g+R_p+R_s/N^2$; $l_1=l_p+l_s/N^2$; $C_1=C_p+N^2C_s$; l_p, primary leakage inductance; l_s, secondary leakage inductance; C_p, primary equivalent shunt capacitance; C_s, secondary equivalent shunt capacitance.

At high frequencies, when winding capacitances have an effect, the equivalent network becomes as shown in figure 3. The high-frequency transfer ratio is given by (ITT 1975)

$$\frac{E_{\text{out}}}{E_g}=\frac{N(R_1+R_2)/R_2}{[(R_1/X_c+X_1/R_d)^2+(X_1/X_c-R_g/R_d-1)^2]^{1/2}}\tag{9}$$

where

$$X_c=1/2\pi f(C_p+N^2C_s)$$

$$X_1=2\pi f(l_p+l_s/N^2).$$

The higher cutoff frequency f_H depends on l_1 and C_1 in the series resonance circuit. If they can be made small, the cutoff frequency increases.

We can reduce the effect of leakage inductance and capacitance by using special constructions and winding methods in the transformer coil design. The most common method is to divide the primary and secondary coils into two sections and place the other winding between the sections (ITT 1975).

There are several phenomena which influence the internal noise of the transformer. One of them is the resistivity of the windings which generates thermal noise. The changes in the permeability of the core in the presence of the internal or an external field generate noise. The cause is usually vibration of the laminations or windings. For this reason it is important to cancel out the static field of the earth. An external noise can be coupled to the transformer as a common mode noise through the capacitive coupling between the primary and the secondary windings or by inductive coupling directly to the windings. An inductive coupling can be reduced by surrounding the transformer with highly conductive or highly permeable metal or by using special coil construction techniques.

4 Construction of the transformer

In this section we discuss an example of transformer construction for a magnetocardiographic measurement system. To obtain as low a cutoff frequency as possible we have made the inductance of the primary winding of the transformer high by using highly permeable core material and thick wire. The core material is soft ferromagnetic metal, so-called mu-metal. The permeability of the core is 11 000 at a frequency of 50 Hz (Vacuumschmelze 1970).

The construction of the transformer and the connection of the windings are presented in figure 4. The windings are divided into two parts to form a differential connection. The secondary voltages E_{e1} and E_{e2} induced by the external magnetic noise flux φ_e are equal and opposite and will be cancelled at the output. The success of the cancellation depends on the balance between the windings. The voltages E_{p1} and E_{p2} induced by the primary flux φ_p are summed at the output.

The leakage inductance and capacitance are reduced by locating the primary coil halves in the middle of the secondary coil halves. This increases the higher cutoff frequency.

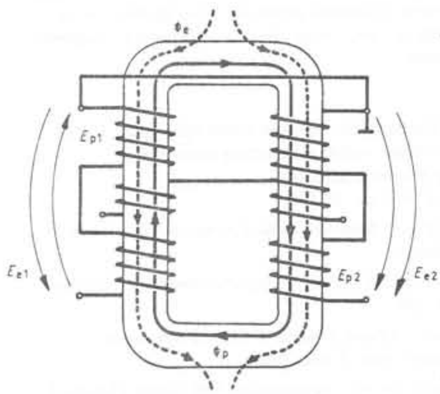

Figure 4 The construction of the transformer and the connection of the windings.

The primary and secondary windings have 400 and 40 000 turns, the diameters of the primary and secondary wires are 0.5 mm and 0.1 mm and resistances are 3 Ω and 32 kΩ respectively.

A thin grounded aluminium foil is placed around the primary winding to eliminate the capacitive coupling. The transformer is covered with a rigid 3 mm thick iron case. This attentuates the external magnetic fields.

5 The amplifier

To obtain very large input impedance, an amplifier with a FET-input stage was chosen. The input bias current is formed by the leakage current of the gate. We have used an instrumentation amplifier LF352 which has a JFET input stage. The input impedance is 2×10^{12} Ω, the input bias current is 3 pA and the input offset current is 0.5 pA.

The gain is 10 000 and is determined with a single resistor. The equivalent input noise current is 10 fA. The equivalent input noise voltage of the amplifier on the bandwidth 0.1–10 Hz is (National Semiconductor Corporation 1978):

$$\sqrt{\overline{e_n^2}} = (1.3 + 670/G)\ \mu V_{pp}, \tag{10}$$

where G is the gain of the amplifier. On the bandwidth 10 Hz–10 kHz the noise voltage is:

$$\sqrt{\overline{e_n^2}} = (8 + 670/G)\ \mu V_{RMS}. \tag{11}$$

Low-frequency or 1/*f*-noise of the amplifier has no effect because of the relatively high lower cutoff frequency of the system.

6 Measured characteristics of the detector system

The frequency response of the transformer was measured with several values of the source resistance. The effect of the source resistance to the response can be seen in figure 5. When the impedance increases the pass band becomes narrower. The resonance frequency of the transformer is 2 kHz. The damping factor is also greatly dependent on the source resistance as can be seen. The best characteristics are obtained with a 50 Ω source resistance. Then the pass band is wide and flat (figure 5).

The electrostatic shielding was tested by measuring the capacitance between the primary and the secondary with a short circuit in the windings (Sommer and Plice 1963). The

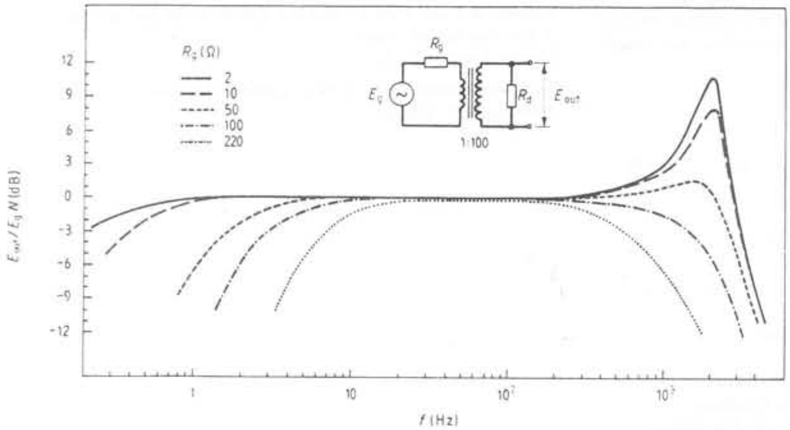

Figure 5 Frequency response of the matching transformer with different values of the source resistance. The load resistance R_d was 10^9 Ω.

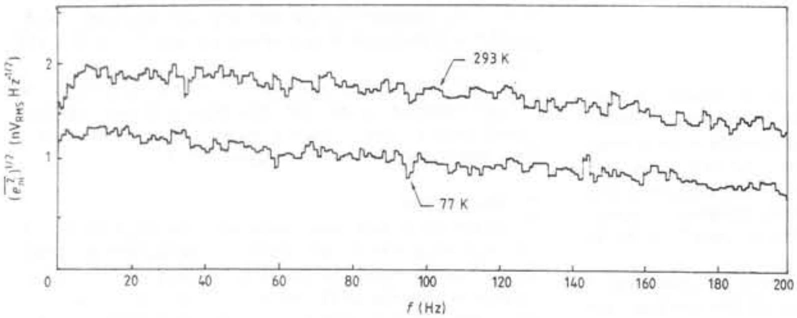

Figure 6 The equivalent input noise $\sqrt{\overline{e^2}_{ni}}$ of the measurement system against frequency at different temperatures of the source resistor. The signal was analysed with Honeywell, Saicor spectrum analyser Model SAI 51B and plotted with XY recorder.

capacitance was 6.4 pF when the aluminium foil shield was not connected to the ground and 3×10^{-4} pF when grounded.

The noise characteristics of the amplifier system composed of the transformer and the amplifier were studied by replacing the detector coil with an equivalent resistance (low noise 120 Ω wire resistor) to eliminate the noise induced by the magnetic field and to preserve the same operating point of the amplifier. The output noise of the system was measured when the resistor was at room temperature (293 K) and when it was cooled with liquid nitrogen (77 K). The output noise was referred to the input of the system (the gain was 10^6) to get the equivalent input noise $\sqrt{\overline{e_{ni}^2}}$. The results are in figure 6. We can see that the equivalent input noise is of the order of 1.8 nV$_{RMS}/\sqrt{}$Hz at room temperature and 1.2 nV$_{RMS}/\sqrt{}$Hz when cooled. By using equation (2) we can calculate the noise figures in both cases. The noise figures are 2.2 dB (293 K) and 4.5 dB (77 K). From equation (1) we can calculate the equivalent input noise generated by the system, the transformer and the amplifier. The equipment noise referred to the input is in the order of 1 nV$_{RMS}/\sqrt{}$Hz.

The amplifier system was used in biomagnetic field detection with an induction coil detector. The effective area of the coil was 80 m^2 and its resistance 120 Ω. The system noise (in temperature 293 K) referred to magnetic field at the measurement location was as low as 180 fT$_{RMS}/\sqrt{}$Hz at 20 Hz.

In the figure 7 we can see a real time recording of the magnetic field of the human heart. The recording was made in a magnetically shielded room (Heinonen *et al* 1980). The detector coil was above the chest of the patient on the anterior side of the body. The distance to the centre of the heart was 80 mm. The noise originates primarily from thermal noise of the magnetometer and from the low-frequency magnetic noise in the shield.

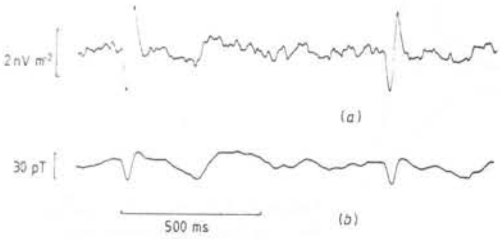

Figure 7 Recording of the magnetic heart vector component from a 26 year old normal male subject. (*a*) The output signal of the coil detector. (*b*) The signal after integration.

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